Hypersonic Interactions with Surface Mass Transfer— Part II: Unsteady Flow

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Abstract

IN this work, the time history of the flow over a slender wedge wing (undergoing a sudden change in angle of attack) until the final steady state is reached has been analyzed.

Contents

The significance of the problem analyzed here lies in determining the control characteristics and devising the effective ways of controlling[1] the surface heating of hypersonic vehicles using wedge-shaped airfoils.

The interaction problems involving impulsive lateral motion of the body are very few in the literature as reviewed in Ref. 2. Here we analyze the unsteady problem with air injection/suction on an inclined plate following an impulsive change in the angle of inclination. The problem of a slender wedge with a sharp leading edge is reduced to that of an inclined plate for the reasons mentioned in Ref. 1 of Part I.[1] The solutions presented here have been obtained by the finite difference scheme of Ref. 3.

The flow governing equations in the transformed plane with surface injection of air can be obtained from the general form of equations given in Part I[1] by taking $\lambda=r=s=Le=c_f=1$. These equations are

$$4x\frac{\partial^2 f}{\partial t\partial\eta}+4x\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial x\partial\eta}-\frac{\partial^2 f}{\partial\eta^2}\frac{\partial f}{\partial x}\right)-f\frac{\partial^2 f}{\partial\eta^2}-\bar{p}\frac{\partial^3 f}{\partial\eta^3}+\beta\left(\frac{2x}{\bar{p}}\frac{\partial\bar{p}}{\partial x}-1\right)\left[H-\left(\frac{\partial f}{\partial\eta}\right)^2\right]=0 \tag{1}$$

$$4x\frac{\partial H}{\partial t}+4x\left(\frac{\partial f}{\partial\eta}\frac{\partial H}{\partial x}-\frac{\partial H}{\partial\eta}\frac{\partial f}{\partial x}\right)-f\frac{\partial H}{\partial\eta}-4\beta\frac{x}{\bar{p}}\frac{\partial\bar{p}}{\partial t}\left[H-\left(\frac{\partial f}{\partial\eta}\right)^2\right]-\frac{\bar{p}}{Pr}\frac{\partial^2 H}{\partial\eta^2}-\frac{2(Pr-1)}{Pr}\bar{p}\frac{\partial}{\partial\eta}\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta^2}\right)=0 \tag{2}$$

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The time-dependent terms have been added to Eqs. (1) and (2). The various symbols have been defined in Part I.[1]

The initial condition in time is specified by the following approach: the steady-state solutions of Part I, corresponding to $\theta_{w,i}=\theta_w$, are assumed to describe the flow on the plate at $t^*<0$. At the instant of impulsive change (i.e., at $t^*=0$) the angle of inclination is incremented by $\bar{\alpha}$. We assume that the entire boundary layer moves with the plate during this change, i.e., relative to the surface, there is no instantaneous change in the spatial distribution of the boundary-layer variables f and H. This is equivalent to saying that the inviscid outer flow adjusts itself instantaneously to the new condition, whereas the flow in the boundary layer, being governed by the viscous processes, takes time to do so.

For solution of the interaction problem, we first introduce, similar to the steady-state problem treated in Part I,

$$\bar{\delta}(x,t)=M_\infty\frac{\delta(x,t)}{\delta_0\bar{\chi}_{L^*}^{1/2}}\,x^{-3/4} \tag{3}$$

where $\delta(x,t)=\delta^*(x^*,t^*)/L^*$ is the boundary-layer thickness.

Using Eq. (3) in the tangent wedge equation,[2] we obtain an equation for $\bar{\delta}$,

$$S(x,t)=K_w x^{1/4}+\frac{3}{4}\xi\bar{\delta}+\xi x\left(\frac{\partial\bar{\delta}}{\partial x}+\frac{\partial\bar{\delta}}{\partial t}\right) \tag{4}$$

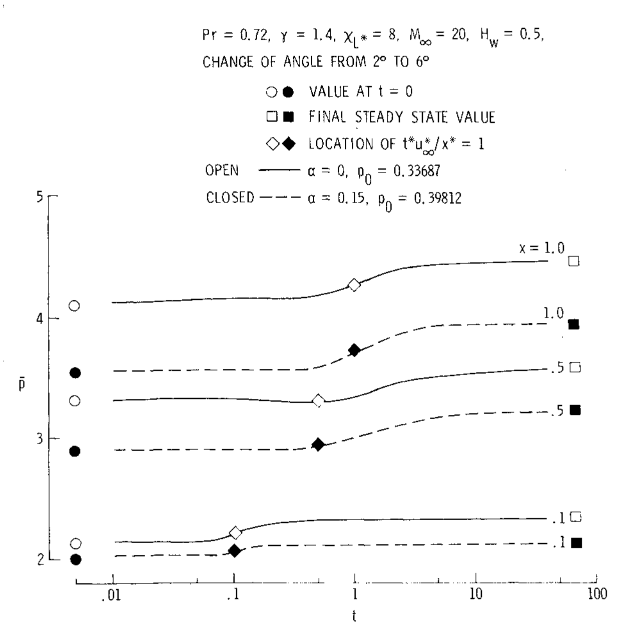

Fig. 1 Variation of induced pressure with time with and without surface injection of air.

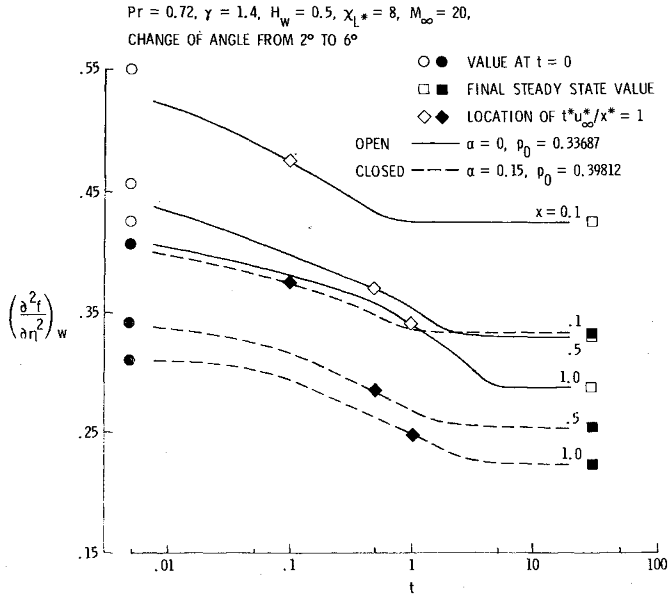

Fig. 2 Variation of wall shear function with time with and without surface injection of air.

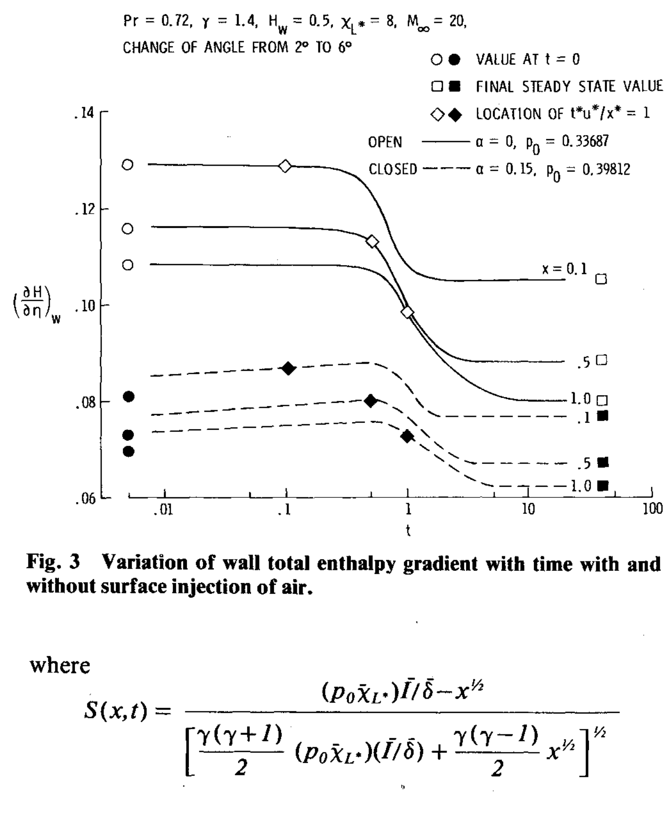

Fig. 3 Variation of wall total enthalpy gradient with time with and without surface injection of air.

where

$$S(x,t) = \frac{(p_0\bar{\chi}_{L^*})\bar{I}/\bar{\delta} - x^{1/2}}{\left[\frac{\gamma(\gamma+1)}{2}(p_0\bar{\chi}_{L^*})(\bar{I}/\bar{\delta}) + \frac{\gamma(\gamma-1)}{2}x^{1/2}\right]^{1/2}}$$

$$\xi = \delta_0 \bar{\chi}_{L^*}^{1/2}$$

The time-dependent term in Eq. (4) may be approximated through the backward-difference form. Thus, we get an ordinary nonlinear first-order differential equation for $\bar{\delta}^{(i)}$ in x for the ith time step. This can be integrated by the fourth-order Runge-Kutta method to obtain $\bar{\delta}^{(i)}$ and $\bar{p}^{(i)}$ can then be computed from[2]

$$\bar{p}^{(i)}(x) = \frac{\bar{I}^{(i)}(x)}{\bar{\delta}^{(i)}(x)}$$

$$\bar{I}^{(i)}(x) = \int_0^{\eta_e}\left\{H^{(i)}(x,\eta) - \left[\frac{\partial f^{(i)}}{\partial \eta}(x,\eta)\right]^2\right\}\mathrm{d}\eta \Big/ I_0 \tag{5}$$

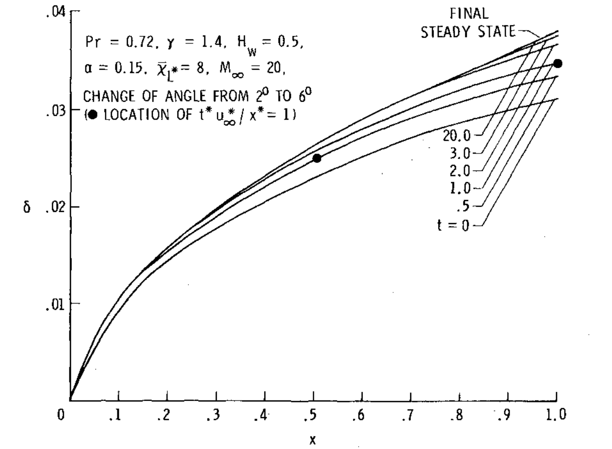

Fig. 4 Distribution of displacement thickness along the plate at different times with surface injection of air.

where I_0 is defined through the zero-order terms of the series expansion solution as[2]

$$I_0 = \int_0^{\eta_e}\left[H_0 - \left(\frac{\partial f_0}{\partial \eta}\right)^2\right]\mathrm{d}\eta \tag{6}$$

Figures 1-3 give the distributions of the induced pressure $p(x,t)$, wall shear function $\partial^2 f(x,0,t)/\partial\eta^2$, and wall gradient of the total enthalpy $\partial H(x,0,t)/\partial\eta$ vs time with ($\alpha \neq 0$) and without injection ($\alpha = 0$). From the values of $\bar{p}$, $\partial^2 f/\partial\eta^2$, and $\partial H/\partial\eta$ given here, the local skin friction coefficient and Stanton number at any instant of time may be computed from the relations given in Ref. 1 of Part I with $\lambda = r = s = Le = c_f = 1$ and $Pr =$ const. The results presented here are for $Pr = 0.72$, $\gamma = 1.4$, $H_w = 0.5$, $\bar{\chi}_{L^*} = 8$, $M_\infty = 20$ deg, and the change in angle of attack is from 2 to 6 deg. The values for initial and final steady states are also indicated in these figures. Further, these figures contain the location of the condition $t^* u_\infty^* / x^* = 1$, which corresponds to the discontinuity $\tau = 1$ quoted in the literature as discussed in Ref. 2. The discontinuity in the present case appears to have been avoided by not employing the variable of type $t^* u_\infty^* / x^*$. In Fig. 1, even though the value of $\bar{p}(x,t)$ with injection ($\alpha = 0.15$) is smaller as compared with the no-injection case, the value of p would be higher because of the somewhat larger value of p_0 in the relation $p = (p_0 \bar{\chi})\bar{p}$.

Figure 4 shows the variation of δ along the plate for different times with injection.

The results presented here indicate that the various quantities approach their respective final steady-state values at $t \approx 10$. The steady state is reached earlier closer to the leading edge.

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